Adaptive Oblivious Transfer with Access Control for Branching Programs

Benoît Libert[1,2] San Ling[3] **Fabrice Mouhartem**[1]
Khoa Nguyen[3] Huaxiong Wang[3]

[1]École Normale Supérieure de Lyon, [2]CNRS,
[3]Nanyang Technological University (Singapore)

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(Adaptive) Oblivious Transfer (OT) [Chau81]

Sender		**Receiver**
DB $(M_1, \ldots, M_N)$		Index ρ
	$\xleftrightarrow{\text{Initialization}}$	
		$(EC_1, \ldots, EC_N)$ $= (Enc(M_1), \ldots, Enc(M_N))$
	$\xleftrightarrow{\text{Transfer}}$	
$\perp$		M_ρ

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- Adaptive OT: receiver adaptively obtains k messages [NP93]
  - Usage: Sensitive DB (DNA, financial data,...).

History

1981 Chaum: introduction

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2009 Camenisch, Dubovitskaya and Neven: access control

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- From FHE + OPRF, or PIR, or ad-hoc pairing assumptions...

No fully simulatable adaptive OT with access control from lattice assumptions

Lattice-Based Cryptography [Ajt96, Reg05]

Lattice

A lattice is a discrete subgroup of $\mathbb{R}^n$. Can be seen as integer linear combinations of a finite set of vectors.

$$\Lambda(\mathbf{b}_1, \ldots, \mathbf{b}_n) = \left\{ \sum_{i \leq n} a_i \mathbf{b}_i \mid a_i \in \mathbb{Z} \right\}$$

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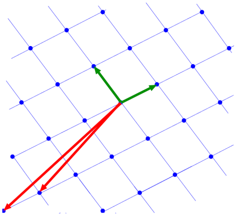


Why?

- Simple and asymptotically efficient;
- **Still** conjectured quantum-resistant;
- Connection between average-case and worst-case problems;
- Powerful functionalities (e.g., FHE).

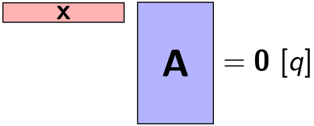
$\rightarrow$ Finding a short non-zero vector in a lattice is hard.

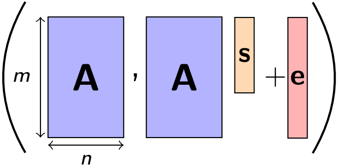
Hardness Assumptions: SIS and LWE [Ajt96, Reg05]

Parameters: n dimension, $m \geq n$, q modulus.

For $\boxed{\mathbf{A}} \hookleftarrow \mathcal{U}(\mathbb{Z}_q^{m \times n})$:

Small Integer Solution	**Learning With Errors**





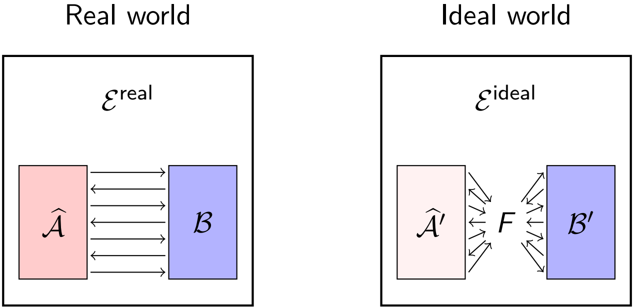
$\boxed{\mathbf{s}} \hookleftarrow \mathbb{Z}_q^n$ $\boxed{\mathbf{e}}$ small error

Goal: Given $\boxed{\mathbf{A}} \in \mathbb{Z}_q^{m \times n}$, find $\boxed{\mathbf{x}} \in \mathbb{Z}^m \backslash \{\mathbf{0}\}$ small

Goal: Given $\left(\boxed{\mathbf{A}}, \boxed{\mathbf{A}}\,\boxed{\mathbf{s}} + \boxed{\mathbf{e}}\right)$, find $\boxed{\mathbf{s}} \in \mathbb{Z}_q^n$

Full Simulation Model [Can01]

For any cheating $\widehat{\mathcal{A}}$, there exists $\widehat{\mathcal{A}}'$ s.t.



$$View(\mathcal{E}^{\mathsf{real}}) \approx_s View(\mathcal{E}^{\mathsf{ideal}})$$

- Strictly stronger security model than indistinguishability-based one

Assisted Decryption Technique [CNs07]

Transfer$_i$

Sender		**Receiver**
		Sample μ at random
		$C_i \leftarrow \mathrm{Rerand}(EC_{\rho_i}, \mu)$
		$= \mathrm{Enc}(M_{\rho_i} \oplus \mu)$
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$+$ Zero-knowledge proofs compatible with the PKE (Keygen, Enc, Dec, Rerand)

Regev's Encryption [Reg05]

Keygen:

Secret key: $\mathbf{S} \hookleftarrow \chi^{n \times t}$

Public key: $(\mathbf{F}, \mathbf{P})$ s.t. $\mathbf{F} \hookleftarrow U(\mathbb{Z}_q^{n \times m})$, $\mathbf{P} = \mathbf{F}^T \mathbf{S} + \mathbf{E}$ with $\mathbf{E} \hookleftarrow \chi^{m \times t}$

Encryption: $(\mathbf{a}, \mathbf{b}) = (\mathbf{a}, \mathbf{S}^T \mathbf{a} + \mathbf{x} + M \lfloor \frac{q}{2} \rfloor) \in \mathbb{Z}_q^n \times \mathbb{Z}_q^t$

Decryption: $M = \lfloor (\mathbf{b} - \mathbf{S}^T \cdot \mathbf{a}) / (\frac{q}{2}) \rceil$

Rerand: $(\mathbf{a}', \mathbf{b}') = (\mathbf{a} + \mathbf{F}\,\mathbf{e}, \mathbf{b} + \mathbf{P}^T \mathbf{e} + \mu \lfloor \frac{q}{2} \rfloor) = Enc(M \oplus \mu)$

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+ ZK proofs

Smudging [AJL+12]

Problem

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Attack scenario:

- $\mathsf{DB} = (M_0, M_1)$
- Sender encrypts M_0 with noise $\mathbf{x}_0$ and M_1 with noise $\mathbf{x}_1$ s.t. $\|\mathbf{x}_0\| \ll \|\mathbf{x}_1\| \leq B_\chi$
- Upon receiving $(\mathbf{c}_0, \mathbf{c}_1)$, decryption leaks $\|\mathbf{x}_i + \mathbf{e}\|$

$\Rightarrow$ sender can break receiver messages' anonymity

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Solution: Flood the noise with ν s.t. $\|\nu\| \gg B_\chi$.

Rerand: $(\mathbf{a}', \mathbf{b}') = (\mathbf{a} + \boxed{\mathbf{F}}\ \mathbf{e}, \mathbf{b} + \boxed{\mathbf{P}}^T \mathbf{e} + \mu\lfloor \frac{q}{2} \rfloor + \nu)$

Access Control [CDN09]

AC-OT

Encrypted database consists in $(EC_i, AP_i)_{i=1}^{N}$.

Receiver can retrieve message M_i iff it possesses cert_x for some $x \in \{0,1\}^*$ s.t. $AP_i(x) = 1$.

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Disjunctions possible through replication.

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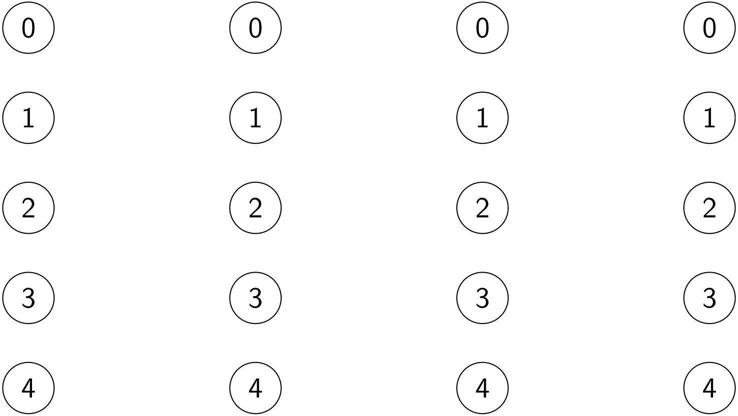
Here: access policy made of branching program (BP).

Branching Programs



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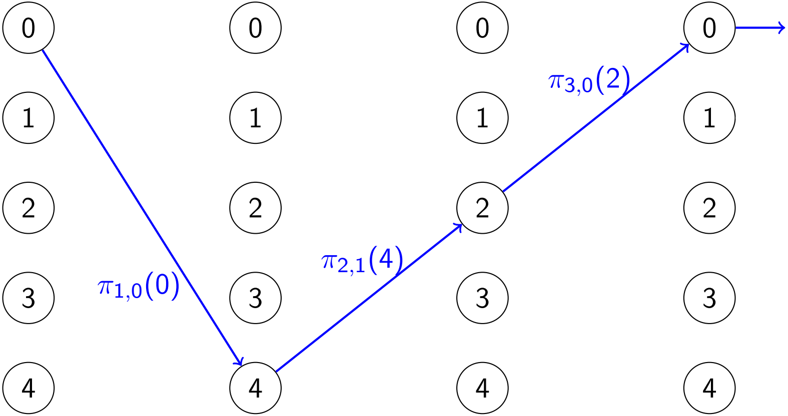
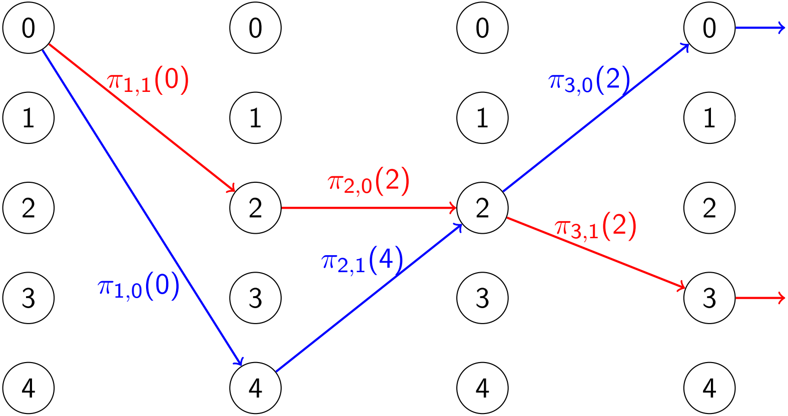


$x = 010$: accepted

Branching Programs



$x = 010$: accepted
$y = 101$: rejected

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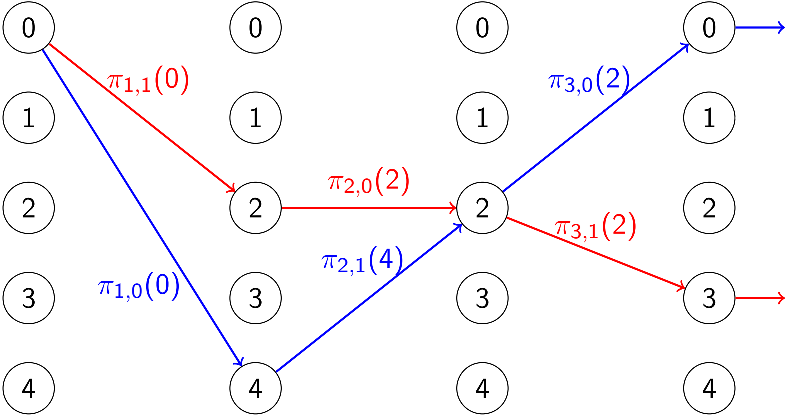

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[Barr86]: polynomially-long BP are equivalent to NC^1

Branching Programs

$$\{(EC_1, BP_1), (EC_2, BP_2), \ldots, (EC_N, BP_N)\}.$$

$$(x, cert_x)$$

Goal: Prove knowledge of $cert_x = Sign(x)$ s.t. $\exists i : BP_i(x) = 1$.

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- Statements over BP interact well with Stern's protocol [Ste93].

Branching Programs

Encoding of a branching program:

$$\mathbf{z}_{BP} = (d_{1,1}, \ldots, d_{1,\delta_\kappa}, \ldots, d_{L,1}, \ldots, d_{L,\delta_\kappa}, \pi_{1,0}(0), \ldots, \pi_{1,0}(4), \pi_{1,1}(0),$$
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$d_{\theta,1}, \ldots, d_{\theta,\delta_\kappa}$: bit representation of $var(\theta)$

Proving correct evaluation

Naively: prove each step $\rightarrow O(L \cdot \kappa)$

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Our idea: use binary-search $\rightarrow O(L \cdot \log(\kappa))$

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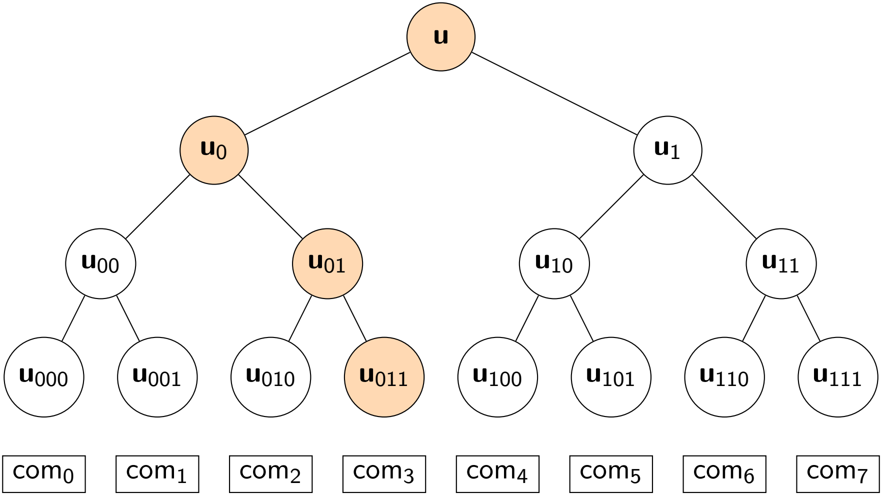

$\mathsf{com}_i = \mathsf{Com}(x_i)$

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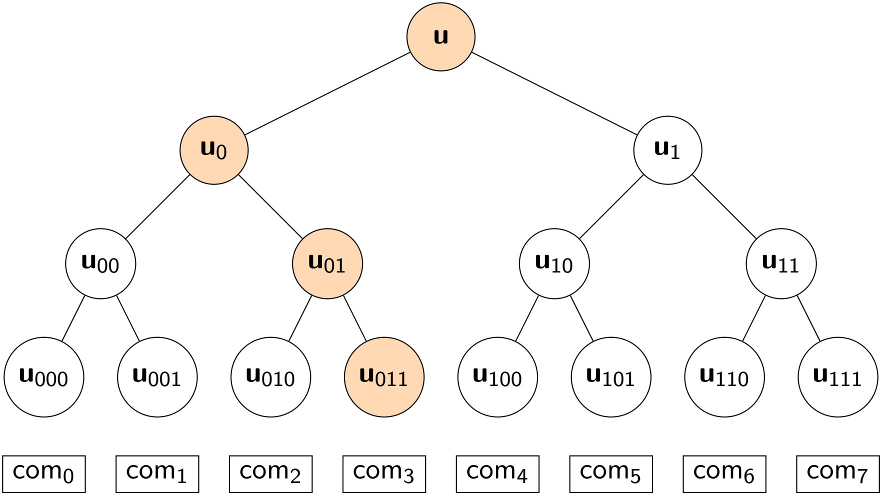

$\text{com}_i = \text{Com}(x_i) \rightarrow$ correct binary search $\Rightarrow$ knowledge of $x_{var(\theta)}$

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Given $\mathbf{P} \in \mathbb{Z}_2^{n \times m}$ and $\mathbf{v} \in \mathbb{Z}_2^{n}$, find $\mathbf{x}$ s.t. $\mathrm{w}(\mathbf{x}) = w$ and

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[KTX08]: mod $2 \to$ mod q

[LNSW13]: Extends Stern's protocol for SIS and LWE statements

Recent uses of Stern-like protocols in lattice-based crypto:
[LNW15, LLNW16, LLN**M**W16]

Our Construction

Ingredients

- The assisted decryption technique
- A simplification of [LLNMW16]'s signature as certificate
- Access control using BP
- WI proofs *à la* Stern

Our Adaptive Oblivious Transfer Construction

Initialization

Sender side:

1. Generate $(\mathsf{VK}_{sig}, \mathsf{SK}_{sig}) \leftarrow \Sigma.keygen(1^\lambda)$
2. Compute $((\mathbf{S}, \mathbf{E}), (\mathbf{F}, \mathbf{P})) \leftarrow Regev.keygen(1^\lambda)$
3. Use $\mathbf{S}$ to compute encryptions of $M_i \rightarrow (\mathbf{a}_i, \mathbf{b}_i) \in \mathbb{Z}_q^n \times \mathbb{Z}_q^t$
4. Use SK_{sig} to compute signatures of $(\mathbf{a}_i, \mathbf{b}_i) \rightarrow \sigma_i$
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5. $EC_i \leftarrow (\sigma_i, (\mathbf{a}_i, \mathbf{b}_i))$

Sender sends $(\mathsf{VK}_{sig}, (\mathbf{F}, \mathbf{P}), EC_i)$ to the receiver and proves that everything was done correctly.

Sender keeps the previous information and $(\mathbf{S}, \mathbf{E})$.

Our Adaptive Oblivious Transfer Construction

Transfer

Sender		Receiver(ρ_i)
		Sample $\mu \leftarrow U(\{0,1\}^t)$ Rerandomize $(\mathbf{a}_{\rho_i}, \mathbf{b}_{\rho_i})$ into $(\mathbf{c}_0, \mathbf{c}_1) = (\mathbf{a}_{\rho_i} + \boxed{\mathbf{F}}\ \mathbf{e},$ $\mathbf{b}_{\rho_i} + \boxed{\mathbf{P}}^T \mathbf{e} + \mu\lfloor q/2 \rfloor \boxed{+\nu})$
	$\xleftarrow{(\mathbf{c}_0, \mathbf{c}_1)}$	
	$\xleftrightarrow{\text{prove knowledge of } \sigma_{\rho_i}}$	
$\widetilde{M}_i = \mathrm{Dec}(\mathbf{c}_0, \mathbf{c}_1)$		
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Final steps

- Access control can be plug over this scheme

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- Access control can be plug over this scheme
- Our scheme is proven secure in the standard model
  - In the ROM: optimizations using NIWI proofs [FS86]

Conclusion

- First AC-OT in the lattice setting that handles expressive statements (NC^1)
- Rely on LWE with superpolynomial modulus
- Proved in the full simulation model

Possible improvements:

- Get rid of the smudging technique

Questions?