

# Structure-Preserving Chosen-Ciphertext Security with Shorter Verifiable Ciphertexts

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3. Preliminaries
4. Construction of Structure-Preserving Publicly Verifiable Encryption

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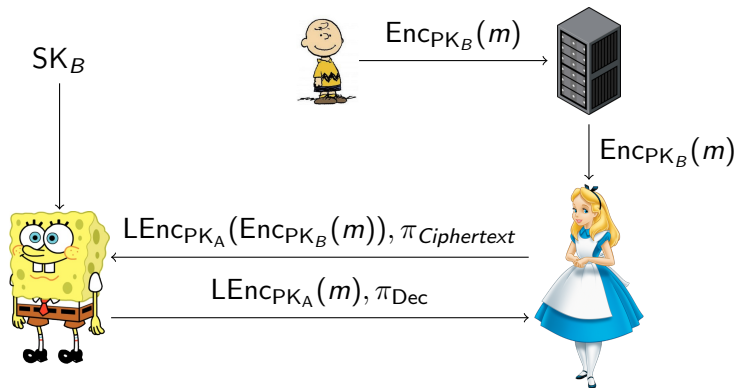
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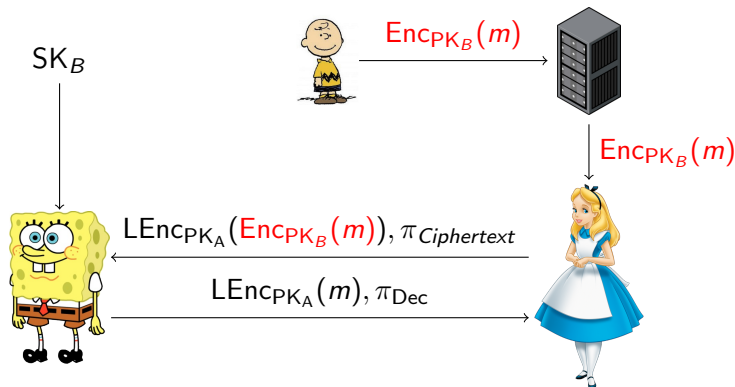
## Example

- 1 Secure blind decryption [Gre11]
- 2 Oblivious 3rd parties protocols [CGH08]

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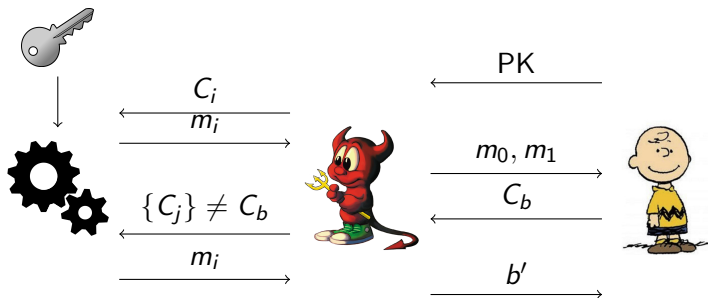


## Public verifiability

- Allows adaptive OT with public contribution to database.
- Allows everyone to check the sanity of the database.
- Makes it possible to distribute senders.

# Indistinguishable Chosen-Ciphertext security (IND-CCA)

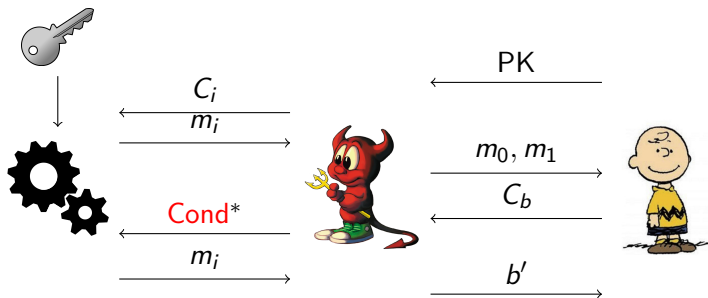
IND-CCA



**Advantage:**  $Adv(\mathcal{A}) = |\Pr[b = b'] - \frac{1}{2}|$ .

# Replayable Chosen-Ciphertext security (RCCA)

RCCA



**Advantage:**  $Adv(\mathcal{A}) = |\Pr[b = b'] - \frac{1}{2}|$ .

- Motivation: Compatible with **perfect rerandomizable** encryption scheme.
- **Optimal** security notion for rerandomizable encryption schemes

---

\*  $Dec(k_d, \{C_j\}) \notin \{m_0, m_1\}$

## Type-3 pairings and DDH (SXDH) assumption

### Pairing

For three groups  $(\mathbb{G}, \hat{\mathbb{G}}, \mathbb{G}_T)$  of prime order  $p$  and  $e : \mathbb{G} \times \hat{\mathbb{G}} \rightarrow \mathbb{G}_T$ .

$$e(A^\lambda, B) = e(A, B^\lambda) \qquad e(g, h) = 1 \text{ iff } g = 1 \vee h = 1$$

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### DDH (SXDH) assumption

Let  $g \in \mathbb{G}$  and  $a, b, c \xleftarrow{R} \mathbb{Z}_p$

- Decisional Diffie-Hellman (DDH):

$$\{g, g^a, g^b, g^{ab}\} \approx_c \{g, g^a, g^b, g^c\}.$$

- Symmetric eXternal Diffie-Hellman (SXDH): DDH in  $\mathbb{G}$  and  $\hat{\mathbb{G}}$ .

# State of the art

## Structure-Preserving Signatures

- [AHO10] Sign a message  $\mathbf{M} = (m_1, m_2, \dots, m_n) \in \hat{\mathbb{G}}^n$  with a signature  $2\mathbb{G} + 5\hat{\mathbb{G}}$  under SXDH with asymmetric pairings.

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## Structure-Preserving Public Key Encryption

- [CHK<sup>+</sup>11] Encryption of a message  $m \in \mathbb{G}$  consists of  $4\mathbb{G} + 1\mathbb{G}_T$  under DLIN with symmetric pairings; not publicly verifiable.
- [ADK<sup>+</sup>13] Structure-preserving publicly verifiable encryption with  $321\mathbb{G}$  under DLIN.

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- [ADK<sup>+</sup>13] Structure-preserving publicly verifiable encryption with  $321\mathbb{G}$  under DLIN.

## Our goals

- Shorter ciphertexts under SXDH in asymmetric pairings (most efficient configuration)
- Public verifiability

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# Contributions

	Ciphertext Size <sup>†</sup>	Assumption	Security
[ADK <sup>+</sup> 13]	$321 \times \mathbb{G}^\ddagger$	DLIN	CCA
This work	$16 \times \mathbb{G} + 11 \times \hat{\mathbb{G}}$	SXDH	CCA
[CKLM12]	$93 \times \mathbb{G}$	DLIN	RCCA
[CKLM12]	$49 \times \mathbb{G} + 20 \times \hat{\mathbb{G}}$	SXDH	RCCA
This work <sup>§</sup>	$29 \times \mathbb{G} + 20 \times \hat{\mathbb{G}}$	SXDH	RCCA

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<sup>†</sup>In the asymmetric setting, we assume  $|\hat{\mathbb{G}}| \approx 2 \cdot |\mathbb{G}|$ .

<sup>‡</sup>Only instantiable with symmetric pairing

<sup>§</sup>Instantiation of their generic construction with the more efficient tools to date.

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Statement: Pairing Product Equation (PPE)

$$\prod_{j=1}^n e(\mathcal{A}_j, \mathcal{Y}_j) \prod_{i=1}^m e(\mathcal{X}_i, \mathcal{B}_i) \prod_{i=1}^m \prod_{j=1}^n e(\mathcal{X}_i, \mathcal{Y}_j)^{\gamma_{i,j}} = t_T$$

Statement: Multi-Exponentiation Equation

$$\prod_{j=1}^n \mathcal{A}_j^{\mathcal{Y}_j} \prod_{i=1}^m \mathcal{X}_i^{b_i} \prod_{i=1}^m \prod_{j=1}^n \mathcal{X}_i^{\gamma_{i,j} \mathcal{Y}_j} = t_T$$

Where for  $i \in \{1, \dots, m\}$  and  $j \in \{1, \dots, n\}$

- 1 Variables:  $\mathcal{X}_i$ ,  $\mathcal{Y}_j$  and  $y_j$ .
- 2 Constants:  $\mathcal{A}_j$ ,  $\mathcal{B}_i$ ,  $t_T$ ,  $\gamma_{i,j}$  and  $b_i$ .

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- Multi-exponentiation equation.
- **Operate in two modes:** Depending on the Common Reference String  
 $CRS = (\mathbf{u}_1, \mathbf{u}_2, \hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2) \in \mathbb{G}^2 \times \mathbb{G}^2 \times \hat{\mathbb{G}}^2 \times \hat{\mathbb{G}}^2$ .

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► **Perfect Zero-Knowledge (ZK) setting**  $\exists \zeta, \hat{\zeta} \in \mathbb{Z}_p$  s.t.  $\mathbf{u}_2 = \mathbf{u}_1^\zeta$  and

$$\hat{\mathbf{u}}_2 = \hat{\mathbf{u}}_1^{\hat{\zeta}}.$$

- 1 Using  $\zeta, \hat{\zeta}$ , we can simulate a proof for false statement.
- 2 Proofs using different valid witnesses are indistinguishable.

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① Using  $\zeta, \hat{\zeta}$ , we can simulate a proof for false statement.

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- ▶ **Perfect Soundness setting**  $(\mathbf{u}_1, \mathbf{u}_2)$  and  $(\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2)$  are independent.

① Even unbounded adversaries cannot prove false statements.

② Trapdoor allows extracting witnesses from proofs.

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- **Correctness:**  $\text{Verify}(\text{ck}, m, \text{Commit}(\text{ck}, m; \text{open}), \text{open}) = \text{True}$ .



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- **Chosen-Message Target Collision Resistant (CM-TCR):** Given  $\text{com}^*, m^*, \text{open}^*$ , hard to generate  $m$  s.t.

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- **Enhanced Chosen-Message Target Collision Resistant (ECM-TCR)**:  
Given  $\text{com}^*, m^*, \text{open}^*$ , hard to generate  $(m, \text{open})$  s.t.

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# Construction ideas: Cramer-Shoup [CS02]

## IND-CCA encryption: Cramer-Shoup

- Keys:  $\text{PK} = g_1, g_2, X = g_1^{x_1} \cdot g_2^{x_2}, \text{SK} = x_1, x_2$ .
- Ciphertext:  $\mathbf{C} = (C_0, C_1, C_2, \pi) = (M \cdot X^r, g_1^r, g_2^r, \pi)$  where  $\pi$  is a proof of  $\log_{g_1}(C_1) = \log_{g_2}(C_2)$  and  $r \xleftarrow{R} \mathbb{Z}_q$ .
- Decryption:  $M = C_0 / (C_1^{x_1} C_2^{x_2})$ .

## Proof intuitions

- Setup:  $\text{PP}_{\text{GS}}$  in perfect soundness setting.
- Challenge Ciphertext:

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# Construction ideas: All-but-one perfectly sound hash proof system [LY12]

## ABO proof

Each proof is associated with a **tag**, prove with  $\mathbf{u}_1 = \mathbf{u}_2 \cdot (1, \frac{1}{tag})$ .

- **Correct tag** GS proof is in perfect soundness setting
- **Wrong tag** GS proof is in perfect ZK setting

Prove  $\log_{g_1}(C_1) = \log_{g_2}(C_2)$

- Generate OTS keys  $VK_{OTS}, SK_{OTS}$ .
- Generate proof of  $\log_{g_1}(C_1) = \log_{g_2}(C_2)$  with  $tag = VK_{OTS}$ .

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(Binding impossible [AHO12])

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- ② Commitment scheme's  $ck$  and  $com$  are in  $\mathbb{G}$  and  $\hat{\mathbb{G}}$ .
  - ▶ **Solution**: No need to sign the commitment. (Not trivial to prove)

# Structure-Preserving Publicly Verifiable Encryption

- $SK = (x_1, x_2)$ .
- $PK = (g_1, g_2, X = g_1^{x_1} g_2^{x_2}, PP_{SPC}, \mathbf{ck}, CRS_{GS} = (\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2) \in \hat{\mathbb{G}}^2 \times \hat{\mathbb{G}}^2)$ .

where  $(g_1, g_2) \in \mathbb{G}^2$ ,  $(x_1, x_2) \in \mathbb{Z}_p^2$  and  $\exists \rho_u \in \mathbb{Z}_p$  such that  $\hat{\mathbf{u}}_1 = \hat{\mathbf{u}}_2^{\rho_u}$ .

## Details of the Encryption algorithm

- Generate  $\mathbf{C} = (C_0, C_1, C_2) = (M \cdot X^\theta, g_1^\theta, g_2^\theta)$ .
- $OTS.KeyGen(PP) \rightarrow (SSK, SVK)$ .
- $Commit(\mathbf{ck}, SVK) \rightarrow (c\hat{o}m, open)$ .
- Compute  $\hat{\mathbf{u}}_{c\hat{o}m} = \hat{\mathbf{u}}_2 \cdot (1, c\hat{o}m) \in \hat{\mathbb{G}}^2$ .
- $Prove(CRS_{c\hat{o}m} = (\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_{c\hat{o}m}), (C_1, C_2), \theta) \rightarrow \pi$  of statement  
 $\exists \chi$  s.t.  $(C_1, C_2) = (g_1^\chi, g_2^\chi)$ .
- $OTS.Sign(SSK, (\mathbf{C}, \pi)) \rightarrow \sigma$ .
- Output the ciphertext  $(\mathbf{C}, \pi, SVK, c\hat{o}m, open, \sigma) \in \mathbb{G}^{16} \times \hat{\mathbb{G}}^{11}$ .

# Proof intuition

## Setup

- $CRS_{GS} = (\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2)$  with  $\hat{\mathbf{u}}_1 = \hat{\mathbf{u}}_2^{\rho_u}$ .

## Encryption

- Generate  $\mathbf{C}^* = (C_0^*, C_1^*, C_2^*) = (M_b \cdot X^{\theta^*}, g_1^{\theta^*}, g_2^{\theta^*})$ .
- $\text{OTS.KeyGen}(\text{PP}) \rightarrow (\text{SSK}^*, \text{SVK}^*)$ .
- $\text{Commit}(\mathbf{ck}, \text{SVK}^*) \rightarrow (\text{côm}^*, \text{open}^*)$ .
- Compute  $\hat{\mathbf{u}}_{\text{côm}} = \hat{\mathbf{u}}_2 \cdot (1, \text{côm}^*) \in \hat{\mathbb{G}}^2$ .
- $\text{Prove}(CRS_{\text{côm}} = (\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_{\text{côm}}), \mathbf{x} = (C_1^*, C_2^*), w = \theta^*) \rightarrow \pi^*$  of the statement " $\exists \chi$  such that  $(C_1^*, C_2^*) = (g_1^\chi, g_2^\chi)$ ".
- $\text{OTS.Sign}(\text{SSK}^*, (\mathbf{C}^*, \pi^*)) \rightarrow \sigma^*$ . **(No need to sign commitments)**
- Output the ciphertext  $(\mathbf{C}^*, \pi^*, \text{SVK}^*, \text{côm}^*, \text{open}^*, \sigma^*) \in \mathbb{G}^{16} \times \hat{\mathbb{G}}^{11}$ .

# Proof intuition

## Setup

- $CRS_{GS} = (\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2)$  with  $\hat{\mathbf{u}}_1 = \hat{\mathbf{u}}_2^{\rho_u}$ .

## Encryption

- Generate  $\mathbf{C}^* = (C_0^*, C_1^*, C_2^*) = (M_b \cdot X^{\theta^*}, g_1^{\theta^*}, g_2^{\theta^*})$ .
- $\text{OTS.KeyGen}(\text{PP}) \rightarrow (\text{SSK}^*, \text{SVK}^*)$ . Done at beginning
- $\text{Commit}(\mathbf{ck}, \text{SVK}^*) \rightarrow (\text{côm}^*, \text{open}^*)$ . Done at beginning
- Compute  $\hat{\mathbf{u}}_{\text{côm}} = \hat{\mathbf{u}}_2 \cdot (1, \text{côm}^*) \in \hat{\mathbb{G}}^2$ .
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- $\text{OTS.Sign}(\text{SSK}^*, (\mathbf{C}^*, \pi^*)) \rightarrow \sigma^*$ . (No need to sign commitments)
- Output the ciphertext  $(\mathbf{C}^*, \pi^*, \text{SVK}^*, \text{côm}^*, \text{open}^*, \sigma^*) \in \mathbb{G}^{16} \times \hat{\mathbb{G}}^{11}$ .

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## Encryption

- Generate  $\mathbf{C}^* = (C_0^*, C_1^*, C_2^*) = (M_b \cdot C_1^{x_1} \cdot C_2^{x_2}, g_1^{\theta^*}, g_2^{\theta^*})$ .
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## Encryption

- Generate  $\mathbf{C}^* = (C_0^*, C_1^*, C_2^*) = (M_b \cdot C_1^{x_1} \cdot C_2^{x_2}, g_1^{\theta_1}, g_2^{\theta_2})$ .
- Compute  $\hat{\mathbf{u}}_{\text{côm}} = \hat{\mathbf{u}}_2 \cdot (1, \text{côm}^*) \in \hat{\mathbb{G}}^2$ .
- Simulate proof of the false statement  $\log_{g_1}(C_1) = \log_{g_2}(C_2)$  using  $\rho_u$ .
- $\text{OTS.Sign}(\text{SSK}^*, (\mathbf{C}^*, \pi^*)) \rightarrow \sigma^*$ . **(No need to sign commitments)**
- Output the ciphertext  $(\mathbf{C}^*, \pi^*, \text{SVK}^*, \text{côm}^*, \text{open}^*, \sigma^*) \in \mathbb{G}^{16} \times \hat{\mathbb{G}}^{11}$ .



## Conclusion

- Publicly verifiable IND-CCA encryption:  $321\mathbb{G} \rightarrow 16\mathbb{G} + 11\hat{\mathbb{G}}$ .
- Publicly verifiable RCCA rerandomizable encryption:  
 $93\mathbb{G}/49\mathbb{G} + 20\hat{\mathbb{G}} \rightarrow 29\mathbb{G} + 20\hat{\mathbb{G}}$ .

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## Future Works

- Smaller ciphertext for rerandomization encryption scheme.
- More general malleability  
(e.g. Linear Homomorphism, HCCA security)?

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